

MODELING UTAH POPULATION DATA

Math 1010 Intermediate Algebra Project

According to data from the U.S. Census Bureau, Population Division, the population of Utah appears to have increased linearly over the years from 1980 to 2008. The following table shows the population in 100,000's living in Utah according to year. In this project, you will use the data in the table to find a linear function $f(x)$ that represents the data, reflecting the change in population in Utah.

	Estimates of Utah Resident Population, in 100,000's					
Year	1980	1988	1993	2000	2004	2009
x	0	8	13	20	24	29
Population, y	14.6	16.9	19	22.4	24.4	27.8

Source: U.S. Census Bureau, Population Division

- Using the graph paper on the last page, plot the data given in the table as ordered pairs.
Label the x and y axes with words to indicate what the variables represent.
- Use a straight edge to draw on your graph what appears to be the line that "best fits" the data you plotted. You will only have one line drawn, rather than several pieces of lines
- Estimate the coordinates of two points that fall on your best-fitting line. Write these points below.

$(18, 22), (3, 15)$

Use the points that you wrote down to find a linear function $f(x)$ for the line.

Show your work!

$$m = \frac{22 - 15}{18 - 3} = \frac{7}{15}$$

$$\frac{15 - b}{3 - 0} = \frac{7}{15} \quad \neq \quad \frac{15 - b}{-15} = \frac{7}{-15}$$

$$-b = \frac{7}{5} - \frac{15 \cdot 5}{1.5} \quad \neq \quad -b = \frac{7}{5} - \frac{75}{5}$$

$$-b = \frac{68}{5} \quad \neq \quad b = \frac{68}{5} = 13 \frac{3}{5}$$

$$f(x) = \frac{7}{15}x + 13 \frac{3}{5}$$

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4. What is the slope of your line? $m = \frac{7}{15}$
 Interpret its meaning. Does it make sense in the context of this situation? Please use complete sentences to respond to these questions.

The slope of my line ($\frac{7}{15}$) implies that the population of Utah increases by approximately 700,000 people every 15 years. Yes, it appears to make sense.

5. Find the value of $f(45)$ using your function from part 3. Show your work, then write your result in the blank below.

$$f(45) = \frac{7}{15}(45) + 13\frac{3}{5} = 21 + 13\frac{3}{5} = 34\frac{3}{5}$$

$$f(45) = \underline{34\frac{3}{5}}$$

Write a sentence interpreting the meaning of $f(45)$ in the context of this project.

In the year 2025 ($1980 + 45$), there should be approx. 3,460,000 people living in Utah if the extrapolated trend continues at its current rate.

6. Use your function from part 3 to approximate in what year the residential population of Utah reached 2,000,000. Show your work.

$$f(x) = 20 = \frac{7}{15}x + 13\frac{3}{5}$$

$$-13\frac{3}{5} \qquad -13\frac{3}{5}$$

$$\frac{15}{7} \cdot 6\frac{2}{5} = \frac{7}{15}x \cdot \frac{15}{7}$$

$$\frac{315}{7} \cdot \frac{32}{5} = x$$

$$\frac{96}{7} = x \rightarrow x = 13\frac{5}{7} + 1980 = \underline{1993}$$

7. Compare your linear function with that of another student or group.

Comparison function: $f(x) = \frac{1}{2}x + 12.4$

Is the comparison function the same as the function you wrote down for part 3?

If they are different, explain why.

This is because the data does not perfectly fit one single function, leaving room for others to fit.

If they are the same, explain why.

8. In actuality, using a linear growth model for population is not common. Most models are exponential models, due to the fact that most populations experience relative growth, i.e. 2% growth per year. Linear models for nonlinear relationships like population work only within a small time frame valid close to the time of the data modeled. Discuss some of the false conclusions you might reach if you use your linear model for times far from 1980-2008.

If we were to use the data in the graph to estimate a relative growth rate over this small window of time we would determine a rate of ~ 28 growth per year, as the question above hints.

Since the same relative proportion of a bigger number is larger than that of a smaller number our linear model is expected to underestimate the population at times long before this period and long after.

long

